is therefore equal to the whole ABCD; but EFCD is a rectangular or right-angled parellelogram, and it has been derived from the given oblique-angled parallelogram ABCD, so that the thing required by the problem is done.

PROBLEM. — *If a parallelogram and a triangle have the same base and are between the same parallels, it is required to prove by the principle of conversion, that the parallelogram is the double of the triangle.*

Example.—Let ABCD (Fig. 3) be a parallelogram and AEB a triangle, subsisting on the same base AB and lying between the same parallels AB and RC; it is required to prove that the parallelogram ABCD is double of the triangle AEB.

Construction.— Reduce the oblique angled parallelogram ABCD, into the equivalent rectangular one ABPF, and in like manner, reduce the triangle AEB into its equivalent rectangular parallelogram AMNF, and the parallelogram ABPF shall be double of the parallelogram AMNF, since the base AB is double of the base AM; but the parallelogram AMNF is equal to the triangle AEB, and ABPF is equal ABCD; hence the equality enunciated in the proposition is already manifest, but to carry out our principle of conversion, we shall throw each of the rectangles ABPF and AMNF into an equivalent square, as this is the figure which we have adopted as the standard of comparison.

On F as a centre with FA as a radius, describe the quadrant AmR, meeting CF produced in R and making FR equal to FA, so that PR is equal to the sum of PF and FA, the sides of the parallelogram ABPF extended in one line. Bisect PR in H, and on H as a centre with HR or HP as a radius, describe the semicircle RGP meeting AF produced in G, and making FG a mean proportional between FP and FA the sides of the rectangle ABPF, and consequently it is the side of an equivalent square.

Again, on F as a centre, with FN as a radius, describe the quadrant NnI meeting AF produced in I and making FI equal FN, so that AI is equal to the sum of FA and FN the sides of the rectangle AMNF extended in one line. Bisect AI in K, and on K as a centre with AK or IK as a radius, describe the semicircle ALI intersecting FP in L; then is FL a mean proportional between FA and FN the sides of the rectangle AMNF, and is therefore the side of an equivalent square. At the point L erect the perpendicular LQ equal to LF, and draw FQ; then since FL is equal LQ, the square of FQ is twice the square of FL; but FQ is equal to FG, for if they be carefully applied to each other they will be found to coincide; hence we infer the specified condition of the figures mentioned; that is, that the parallelogram is double of the triangle.

PROBLEM.—*It is required to prove by the principle of conversion, that in every parallelogram, the complements of those parallelograms which are about the diagonal of the given parallelogram are equal to one another.*

Example.—Let ABCD (Fig. 4) be a parallelogram of which the diagonal is BD, the complements of the parallelograms which are about the diameter being AEHN and CFHI; it is required to prove that these complements are equal to one another.

Construction.— Convert the oblique parallelograms AEHN and CFHI into the equivalent rectangular ones EHPR and FHTM; then on H as a centre with the radii HE and HF, describe the circular quadrants EmS and FnQ meeting HT produced in S and HP produced in Q, so that HS and HQ shall be respectively equal to HE and HF; then is PS the sum of the sides EH and HP extended in one line, and QT is the sum of the sides HF and HT similarly extended. Bisect PS in L and QT in K, and on K and L as centres with radii KQ and LP describe the semicircles QGT and PGS intersecting EH produced in G; then is HG a mean proportional between HQ and HT, and is consequently the side of a square which is equivalent to the rectangle HFMT; but HG is also a mean proportional between HE and HP, and is therefore the side of a square equivalent to the rectangle EHPR; these two rectangles are therefore equal; but they are equal to the complements AEHP and HFCI; hence the equality of these complements is manifest.

PROBLEM.— *To construct a parallelogram upon a given straight line, that shall be equal in area to a given triangle, and have an angle equal to a given angle; that is, to convert a given triangle into a parallelogram of equivalent area, having one of its sides and one of its angles given.*

Example.—Let *ab* (Fig. 5) be the given straight line, DEF the given angle, and ABC the given triangle; it is required to constitute a parallelogram on the straight line *ab*, that shall be equal in area to the triangle ABC, and have an angle equal to DEF.

Construction.—Bisect AC, the base of the given triangle in L, and at the point L make the angle CLM equal to the given angle DEF; then through C, the extremity of the base AC, of the given triangle ABC, draw the straight line CP parallel to LM, and through B, the vertex of the triangle ABC, draw the straight line BI parallel to AC, the base of the triangle, and intersecting LM and CP in the points M and P. Produce LM and CP directly forward to G and N, making PN equal to *ab*, the given line; through the point N draw the straight line GNH parallel to BI, and join GP, which extend directly forward to meet AC, the base of the triangle produced, in K, and through K draw KIH parallel to CPN, and meeting BI and GH in I and H; then is PNHI the parallellogram required; being equal in area to the given triangle ABC,—having an angle equal to the given angle DEF, and a side PN, or IH equal to the given line *ab*.

Demonstration.—Because AC the base of the triangle ABC, is bisected in L, the triangle ABL is equal to the triangle LBC, and consequently the whole triangle ABC is double of the triangle LBC; but the parallelogram LMPC is double of the triangle LBC, and for this reason the parallelogram LMPC is equal to the triangle ABC, and it has by construction, the angle CLM equal to the given angle DEF. But the parallelogram PNHI is equal to the parallelogram LMPC, being complements about the diameter GK, and it has the angle NPI equal to CLM, and moreover it is applied to the line PN, which is equal to the given line *ab*; but it has been shewn that LMPC is equal to the triangle ABC; hence also PNHI is equal to the triangle ABC, and as the other conditions of the problem have been shown to be satisfied, the thing required is done.

PROBLEM.— *To construct a parallelogram that shall be equal in area to a given four-sided right-lined figure, and have one of its angles equal to a given angle; that is, to convert a given right-lined figure into a parallelogram of equal area, having one of its angles given.*

Example.—Let ABCD (Fig. 6) be the given right-lined figure, and EFG the given angle; it is required to convert the given right-lined figure ABCD into a parallelogram of equal area, and having an angle equal to the given angle, EFG.

Construction.— Draw the diagonal AC, dividing the given figure ABCD into the two triangles, ABC and ADC, and divide AC into two equal parts in the point P. At the point P, make the angle CPH equal to the given angle EFG, and produce HP in the opposite direction to M. Through C, the extremity of the common base AC, draw the straight line LCK parallel to MH, and through the opposite angles, B and D, draw the straight lines BK and ML parallel to AC the diagonal, and to each other, and meeting MH and LK in the several points, M, H, and L, K, thereby forming the parallelogram MHKL equal to ABCD, the given figure, and having an angle equal to EFG, the given angle.

Demonstration.—Because the diagonal AC is bisected in the point P, the whole triangle ABC is double of the triangle PBC; but the parallelogram PHKC is double of the triangle PBC, since it stands upon the same base, PC, and is between the same parallels, PC and BK; consequently, the parallelogram PHKC is equal in area to the triangle ABC, and it has the angle CPH equal by construction to the given angle, EFG.

Again, since AC is bisected in the point P, the whole triangle ADC is double of the triangle PDC; but the parallelogram PMLC is double of the triangle PDC, since they are upon the same base PC, and between the same parallels PC and ML, therefore the parallelogram PMLC is equal to the triangle ADC, and it has the angle LMP, equal by construction to the given angle EFG. But the two triangles, ABC and ADC, are together equal to the given figure ABCD; hence, also the two parallelograms PHKC and PMLC are together equal to the given figure ABCD, so that the given figure has been converted into a parallelogram of equal area, and having an angle equal to the given angle EFG, according to the conditions of the problem.

Note.—EUCLID performs this problem differently, but not so simply and elegantly as by means of the foregoing; his mode of construction is as follows:—draw the diagonal AC, dividing the figure into the two triangles ABC and ADC, and bisect the diagonal AC in the point P; then, upon half the diagonal PC, constitute the parallelogram PHKC equal to the triangle ABC, and having an angle equal to the given angle EFG. To the straight line PC apply the parallelogram PMLC, equal to the triangle ADC, and having an angle equal to the given angle EFG; then is the whole parallelogram MHKL equal in area to the given figure ABCD, and having an angle equal to the given angle EFG, the same as before. The preceding is the general method of solution; but when the given angle is a right angle, the operation is somewhat simplified, and the resulting parallelogram is better adapted for practical purposes.

EARLY AND RIGHT-DIRECTED PATRONAGE TO ARTISTS.

SIR,—In your last week's paper I saw a succinct account of the early career of Gibson in his profession, and what struck my attention was, that while he was unknown to fame, and obscure (as every genius must be in the beginning, until some kind hand is stretched forth to bring him into notice), Mr. Roscoe came forward, and was not only his early patron, but was the means of procuring a succession of munificent patrons. Surely, under such an auspicious beginning, any man that had abilities for his profession must have prospered. Give a "sound" vessel but fair wind and deep water, and you may be certain of a prosperous voyage. I consider, sir, the commencement of Gibson's career as a sculptor an auspicious one indeed; and I would here observe, that the most important part of an artist's life is the beginning; and it may not be out of place if I further observe, that all those who wish well to art and its cause, could not confer a greater blessing upon it than to direct attention to the able who are obscure and for the first time send their works for exhibition.

Attention thus directed might be the cause of not only aiding art, but of rescuing many (who have spent the best part of their lives in arduous study) not only from disappointed hope, but despair and an untimely grave. It is a strange matter of fact, that many who have devoted themselves to this profession have more or less been martyrs for the cause. A score of such cases occur as the year rolls round, that the world knows nothing of, because they are obscure. Little do the thoughtless think (when they are gazing upon some painting) of the weary hours, and it may be at last, the disappointed hope, which has driven the author of it to despair and an untimely grave. It is true he may have produced a creditable piece of work, and sent it forth to the world; but the world had contented itself by looking to those only which were familiar to it, and well known to fame. The critic, too, might pass it with indifference; or if he did stop to say any thing, it might be to cast upon it some thoughtless brand. In such a case it is no wonder,—in fact, a man must be more than human if he did not sicken under this state of things.

Now, had such a one received the timely aid, such as Roscoe gave to Gibson, there would not only have been one saved from an untimely grave, but, through his talents, a blessing conferred on generations yet unborn. Now mark the inconsistency (it may be an "oversight") of the world,—let us hope so. It may be that such hath lain in his grave three, four, or a score of years, when the value of his painting becomes appreciated; then there is offered for it not only the amount which he asked for it (which would have saved him from a world of woes), but twenty, ay, forty times the amount. But what good does this now? Nay, is it not a mockery to the dead? Can *that* bring back life? Can it

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